NOTATION

d, particle diameter; m, number of blades; m_p, particle mass; n and s, generalized coordinate axes; $\dot{n}$ and $\dot{s}$, generalized velocities; n_r, number of rotor revolutions; w_0 and w(r), gas velocities at the inlet to the channel and the instantaneous velocity of the radius; v_n and v_s, projections of particle velocities onto the n and s axes, respectively; $|v_n|$, projection of particle velocity onto the n axis in terms of absolute magnitude; R_0 and R_i, outer and inner wheel radii; y and x, nonmoving coordinate axes; T, kinetic energy of the particle; Q_n and Q_s, projection of the generalized forces onto the n and s axes, respectively; ω, angular velocity; ψ, resistance factor; ρ_p and ρ_g, density of particle and gas, respectively; μ_g, gas viscosity; $Re = w_0 d \rho_g / \mu_g$, Reynolds number.

LITERATURE CITED

1. B. Brettschneider and J. Kurfuerst, *Protecting the Water Basin from Contamination. Technology and Control* [Russian translation], A. F. Tubolkhin (ed.), Leningrad (1989).
2. P. A. Kouzov, A. D. Mal'gin, and G. M. Skryabin, *Removal of Dust from Gases and Air in the Chemical Industry* [in Russian], Leningrad (1982).
3. N. N. Zhenishek, *Rotational Dust Removal* [in Russian], Moscow (1958).
4. "Centrifugal separator," USSR Inventor's Certificate, No. 521,911: MKI^2 V 01 D 45/12.
5. "Centrifugal foam extinguisher," USSR Inventor's Certificate, No. 578,330: MKI^2 S 12 V 1/18.
6. S. M. Targ, *A Brief Course of Theoretical Mechanics* [in Russian], Moscow (1967).
7. A. G. Kasatkin, *Fundamental Processes and the Apparatus of Chemical Technology* [in Russian], Moscow (1971).

USING THE REVERSABILITY OF THE PELTIER EFFECT TO REDUCE THE HEAT-SCATTERING SURFACES OF THERMAL COOLING BATTERIES

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We propose and analyze a method for significant reduction in the area of heat scattering surfaces in thermal cooling batteries, where the latter are used to cool small-scale objects.

In view of the increasing scale on which thermoelectric cooling batteries are used presently to cool small-scale objects in radio- and microelectronics, considerable importance has been ascribed to the dissipation of heat from the hot junctions of thermal batteries. In engineering practice, with this purpose in mind, use is often made of radioelectronic equipment (REE) design elements, and here, in a number of cases, owing to the small dimensions of the apparatus itself, such surfaces turn out to be overheated, and this in turn leads to an elevation in the temperature of the object to be cooled, i.e., to a reduction in the efficiency of the cooling process.

In the present study we propose and analyze a method of activating the heat-scattering process by altering (elevating) the temperature of an artificially generated heat-scattering surface of smaller area, while retaining a lower temperature in the object being cooled. The basic diagram of the proposed method is illustrated in Fig. 1. In Fig. 1a we see illustrated the situation which actually prevails in the cooling of REE elements. The REE element is cooled by means of a single-cascade thermal battery 2, removing heat to the bounded upper surface of the device (a radiator), and here, owing to the inadequate size of the heat-scattering surface 1 and 3, a rather substantial drop in temperature is produced between the radiator and the ambient medium, which in the final analysis reduces the capabilities of the REE element. Within the scope of the proposed method, this same surface (Fig. 1b) is connected to the second thermal cooling battery 4 through its cold junction. Its hot junction in this case is oriented into open space and fitted with radiator 5 to dissipate heat at a higher temperature potential. If it is possible to prove that the area of radiator 5 in such a scheme is significantly smaller than heat-scattering surfaces 1 and 3, the problem may be regarded as having been solved. Particular attention should be devoted here to the fact that the smaller area 5 replaces the larger area 3, thus achieving the same (and possibly a lower) level of cooling for the REE object.

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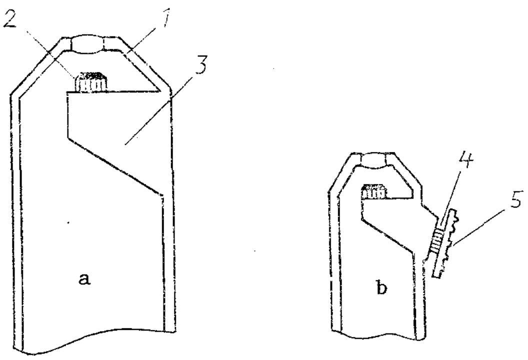

Fig. 1. Diagram showing the cooling of the REE element of a semiconductor thermoelectric battery: a) existing version; b) proposed version; 1, 3) structural elements designed to scatter the heat generated by the thermal batteries; 2) an REE element with cooled thermal battery; 4) thermal battery of the reference cascade state; 5) "high-temperature" radiator-heat scattering elements.

TABLE 1. Ratio of Scattering Radiator Surfaces for Various Cases of Heat Removal

$T_2 - T_m$ (radiator-medium), deg	5	5–10	10–20	20–30	30–40	40	50	60
ε_2	—	8,3	2,5	1,4	1,15	0,96	0,62	0,44
R	1	1,2	1,6	2,0	2,2	2,3	2,18	2,13

Prior to our formulating a mathematical model of the problem, from a qualitative standpoint let us examine each of the heat-removal schemes. In the former case (a single cooling cascade with removal of heat to the structural element) the dissipation of heat proceeds with a relatively slight temperature difference between the structural element and the medium (5-10°), which (the difference) nevertheless must be regarded as a "spurious difference," since it increases the overall temperature difference across the thermal battery, reducing its cooling coefficient. In this event the removal of heat comes about as a consequence of the free convection of air and through thermal insulation, with both processes occurring with a limited temperature difference. In the version proposed here, the second cascade (we will refer to it here as the reference cascade) generates an elevated temperature at the radiator, as a consequence of which there is a sharp increase (proportional to the new temperature difference between the radiator and the medium) in the convective removal of heat and an even greater increase in the case of radiative heat removal. It is significant, however, to note that in such a system it is not only the heat removed from the object being cooled that is carried away, together with the thermal equivalent of the electrical power of the cooling thermal battery, but the additional heat, as well, from the reference cascade.

In such a situation it is only quantitative estimates that can give answers to the question as to whether or not the proposed method is useful.

Using the basic relationships from the theory of energy applications of thermal cooling elements [1], let us examine the formulated problem, introducing the following notation: Q_0, the thermal load on the cold stage; ε_1, the cooling coefficient of the cold cascade; W_1, the electrical power of the cold cascade; Q_1, the heat evolved at the external (hot) junction of the cold cascade (however, it serves as the heating load on the reference cascade); ε_2, the cooling coefficient of the reference cascade; W_2, the electrical power of the reference cascade; Q_2, the heat released at the hot junction of the reference cascade; S_1 and S_2, the heat-scattering surfaces of the cold and reference cascades, respectively; η_c, coefficient of convective heat transfer; β, the heat-removal surface emissivity; T_m, temperature of the ambient medium; T_1, temperature of the object being cooled; T_0, temperature of the contact surface for the cold and reference cascades; T_2, temperature of the hot junction in the reference cascade (the "high-temperature" radiator); $T_0 - T_m$ and $T_2 - T_m$, temperature differences across the medium and the radiators in each of the cases examined here; γ, a coefficient in the Stefan–Boltzmann law; Z, average thermal electrical efficiency of the thermoelectric materials of the p and n types, making up the basic thermal battery.

Thus we have: $Q_1 = Q_0 + W_1$, where $W_1 = Q_0/\varepsilon_1$.

Then

$$Q_1 = Q_0\left(1 + \frac{1}{\varepsilon_1}\right); \quad Q_2 = Q_1 + W_2, \tag{1}$$

and here $W_2 = Q_1/\varepsilon_2$.

Using (1), we obtain

$$Q_2 = Q_0 \left(1 + \frac{1}{\varepsilon_1}\right)\left(1 + \frac{1}{\varepsilon_2}\right). \tag{2}$$

Let us introduce the radiative and convective heat-exchange terms:

$$Q_c = \eta_c S (T - T_m); \; Q_{rad} = \beta\gamma S (T^4 - T_m^4).$$

For the sake of simplicity we will assume that $\beta = 1$. As a result,

$$Q_{tot} = S \{\eta_c (T - T_m) + \gamma (T^4 - T_m^4)\}, \tag{3}$$

where T is the temperature of the heat-scattering surface. Then

$$Q_1 = Q_{tot}^{I} = S_1 \{\eta_c (T_1 - T_m) + \gamma (T_1^4 - T_m^4)\},$$
$$Q_2 = Q_{tot}^{II} = S_2 \{\eta_c (T_2 - T_m) + \gamma (T_2^4 - T_m^4)\} \tag{4}$$

(I, the cold cascade; II, the reference cascade). Into the left-hand side of system of equations (4) we will substitute the values of Q_1 and Q_2 from (1) and (2) and we will determine the magnitude of the ratio S_1/S_2 as a function of the thermal parameters of the heat-scattering surfaces and the energy parameters of two-cascade thermal batteries:

$$\frac{Q_1}{Q_2} = \frac{S_1 \{\eta_c (T_1 - T_m) + \gamma (T_1^4 - T_m^4)\}}{S_2 \{\eta_c (T_2 - T_m) + \gamma (T_2^4 - T_m^4)\}} = \frac{1}{\left(1 + \frac{1}{\varepsilon_2}\right)}.$$

From this we find that

$$\frac{S_1}{S_2} = \frac{\{\eta_c (T_2 - T_m) + \gamma (T_2^4 - T_m^4)\}}{\left\{\eta_c (T_1 - T_m) + \gamma (T_1^4 - T_m^4) \left(1 + \frac{1}{\varepsilon_2}\right)\right\}}, \tag{5}$$

where ε_2 is defined by the familiar formula

$$\varepsilon_2 = \frac{T_0}{T_2 - T_0} \, \frac{\sqrt{1 + \bar{Z}\frac{T_0 + T_2}{2}} - \frac{T_2}{T_0}}{\sqrt{1 + \bar{Z}\frac{T_0 + T_2}{2}} + 1}. \tag{6}$$

As we can see, Eq. (5) is independent of Q_0 and ε_1.

Relationship (5) is decisive in the proposed heat-removal scheme and we will therefore briefly clarify its physical significance. The quantity S_1/S_2 shows how the radiator accomplishing the removal of a quantity of heat $Q_0 + W_1$ to the surrounding medium in the case of a small (units of degrees) temperature difference relates to the radiator removing a substantially larger quantity of heat $Q_0 + W_1 + W_2$, but with a significantly elevated temperature difference between the radiator and the medium (tens of degrees). The case $S_1/S_2 = 1$ shows identical economies for both heat-removal methods. The case $S_1/S_2 > 1$ indicates that the second case of heat removal can be realized with a smaller radiator area.

Once we have completed the double differentiation operation in expression (5) with respect to $(T_2 - T_m)$ and $(T_2 - T_0)$ and after we have completed the corresponding transformations, it should be possible to find the quantity $(T_2 - T_m)$ at which S_1/S_2 attains its optimum, which would be the correct solution for the formulated problem. However, this mathematical operation is made difficult by reason of the various laws governing the dynamics of the quantities $(T_2 - T_m)$ and $(T_2 - T_1)$, the relative complexity of expression (6) itself, and the high temperature levels in the "radiation" term of expression (5).

We will make an attempt to solve a simpler problem, namely we will undertake a qualitative analysis of expression (5) and examine a version of its numerical solution for specified quantities T_0 and T_m and discrete quantities $(T_2 - T_m)$, $(T_2 - T_0)$, and T_2. Having analyzed expression (5), it is not difficult to determine its extreme points for $(T_2 - T_m) = (T_1 - T_m)$, i.e., at the temperature difference across the junctions of the reference cascade, equal to zero: $S_1/S_2 = 1$. With $(T_2 - T_0)$, as it approaches ΔT_{max} for the reference cascade, the value of ε_2 tends toward zero and S_1/S_2 also tends toward zero, i.e., the heat-removal surface must increase without limit. Naturally, neither of these cases are of practical interest. However, the significantly different nature of the relationship between ($T_2^4 - T_m^4$) and ε_2 with respect to the temperature difference across the junctions of the reference cascade shows that at low temperature differences the numerator in expression (5) will grow more rapidly than the denominator (i.e., $S_1/S_2 > 1$), which characterizes the technical suitability of the process in which the removal of the heat generated by the system can be accomplished at a smaller radiator surface S_2. However, with significant values for ($T_2 - T_m$) as a consequence of a sharp rise in ($T_2 - T_0$) and a corresponding reduction in ε_2, the quantity S_1/S_2 begins to drop and finally must become smaller than unity, which makes the utilization of the reference cascade uneconomical.

Let us examine this dynamics on a specific example and let us make an attempt to determine an optimized magnitude of S_1/S_2 for specified computational parameters.

It should be noted that from the standpoint of "classical" theory in connection with energy applications of thermal cooling batteries, the formulated problem would be adequate to achieve a maximum difference across the junctions of a two-cascade thermal battery with a relative minimizing of ($W_1 + W_2$), i.e., the total energy consumption of the cascades. We know that in first approximation [2] this is achieved with equality of the cooling factors for the two cascades ($\varepsilon_1 = \varepsilon_2$). However, in our case this may not be realized, owing to the significantly different sensitivity of the reference-cascade cooling factor to the magnitude of ($T_2 - T_m$) and on the part of the "radiation term" in the numerator of expression (5). Moreover, it follows directly from expression (5) that it is independent of ε_1.

We will assume the following parameters for the numerical calculations: $Z_{n,p} = 2.5 \cdot 10^{-3}$ deg^{-1}, $\eta_c = 5$ W/(m^2·K), $\gamma = 5.7 \cdot 10^{-12}$ W/(m^2·K^4), ($T_1 - T_m$) = 5°, $\beta = 1$. The calculation will be carried out with increments in ($T_2 - T_m$) of 5°. For the initial temperature of the medium we will assume 300 K. The first theoretical point for the radiator of the cooling cascade will be positioned at ($T_1 - T_m$) = 5°, while for the reference cascade we will have $T_2 = T_1$ and, according to (5), $S_1/S_2 = 1$. We will denote S_1/S_2 as R.

Table 1 shows the results of the numerical calculation. It follows from this table that the rather shallow maximum in S_1/S_2 is situated in the range of temperature differences 35-45° between the radiator and the medium, subsequent to which R begins slowly to drop. From the standpoint of technical applications it is possible (for the given computational case) to use a slightly lower difference (for example, 25-30°). In this case, with a slight loss in R (up to 10%) we achieve a gain in the required electrical power (see the data for ε_2 in Table 1).

The physical sense of the relationships indicate, for example, that with a temperature difference of 40° for the radiator and the medium and the release at this radiator of the thermal power of a two-cascade thermal battery, the area of the latter can nevertheless be smaller by a factor of 2-3 than the surface of the original radiator, such as that used only by a single-cascade thermal battery at a temperature difference of 5°C between the radiator and the medium. The temperature of the cold junction (i.e., of the object) in each of the cases here will be identical.

We should take note of yet another circumstance which is significant. The structural positioning of the reference cascade predetermines the presence in that cascade of a "hot surface, turned to the outside," with a high-temperature scattering radiator positioned above that surface (see Fig. 1b). Such a disposition makes it possible for both convective and radiative heat removal to use each of the radiator surfaces (more precisely, to use the surface of the second radiator, excluding the surface areas of the hot thermal-battery junction). Thus the numerals 2 and 3 are changes to 4 and 4.5, i.e., the surface of the heat-scattering radiator can be reduced by factors of that extent. Apparently, the indicated rather sharp reduction in area may be of practical interest in a number of cases.

Objectivity demands that we note the total energy required in this case increases for the unit by a factor of virtually two (see Table 1, where $\varepsilon_2 = 1.15$); however, in a number of applications the alternative to a reduction in the heat-removal area with simultaneous increase in the required power will be uniquely resolved in favor of the first possibility.

LITERATURE CITED

1. A. K. Ioffe, L. S. Stil'bans, E. K. Iordanishvili, and T. S. Stavitskaya, *Thermoelectric Cooling* [in Russian], Moscow (1956).
2. A. I. Burshtein, *Physical Fundamentals for the Design of Semiconductor Thermoelectric Devices* [in Russian], Moscow (1962).